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A GENERAL ADAPTIVE CONTROL STRUCTURE

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ABSTRACT

A general adaptive control structure is given which provides flexibility in defining the plant and controller parameterizations, and the goal of the adaptation. A special case is conventional model reference adaptive control with independent uncertainty in all of the plant transfer function coefficients, although a much broader class of systems can be represented. A single derivation of the error equations for parameter estimation purposes is provided, and is valid for the entire class of systems. The form of the resulting error equations is appropriate for application of common parameter estimation techniques.

I. INTRODUCTION

An adaptive control structure is the collection of (1) a plant description containing uncertain parameters, (2) a controller description containing adjustable parameters, and (3) a design rule or "goal" which maps plant parameters into controller parameters.

This paper presents a general adaptive control structure in an especially simple form. Each of the three system elements (plant, control, goal) is represented by a linear equation. A linear constraint assures that the goal is reachable for all possible values of the plant parameters, by suitable choice of the controller parameters. A standard linear parameter estimation problem results for all represented systems, forming an interface to common parameter adjustment mechanisms.

In spite of its simplicity, the representation of this paper is flexible. Specifically:

(A) The representation provides significant freedom in specifying the location of the plant's uncertain parameters. A special case of the representation is the frequently-studied case of independent uncertainty in all of the plant transfer function polynomial coefficients. However, more generality is allowed here. The unknown parameters may be imbedded in the plant dynamics, and the number of unknown parameters need not correspond to the order of the plant transfer function. Nonparametric (unstructured) uncertainty is allowed as well.

(B) The representation provides significant freedom in specifying the goal of adaptation. The much-studied case of model reference adaptive control is suppported, as well as loop-shaping adaptive control, and a host of other yet-unexplored alternatives.

The flexibility afforded by the general structure representation should prove to be important in the developement of adaptive control structures with desirable tuned system performance and robustness properties. A single convenient representation for a broad class of systems should expedite the comparison of alternative structures.

The organization of the paper is as follows. Section II provides some notation. Section III describes the general structure in two steps. First the ideal tuned system equations are given. Second, the actual untuned system equations are given, with additional nonparametric uncertainty included. Section IV contains a single derivation of the parameter estimation problem, valid for all systems representable by the general structure. The resulting standard linear error equation mates well with conventional estimation algorithms, such as recursive least-squares. With nonparametric uncertainty included, the estimation problem corresponds exactly to that addressed by the robust parameter adjustment theory of [4], [2]. Section IV provides special cases of the plant, control, and goal equations.

II. NOTATION AND DEFINITIONS

A. Notation

Due to the nonlinear and time-varying nature of adaptive control systems, it is necessary to perform the system analysis in the time domain. However, Laplace representations of Linear Time-Invariant (LTI) operators are convenient. Thus, throughout the paper, signals are time-domain quanities and all operations are time domain operations. Thus, even if an operator T is described in the Laplace domain, when applied to an input u, as in $y = Tu$, it is to be understood as a time domain convolution operation.

For a polynomial

$$G(s) = g_n s^n + g_{n-1} s^{n-1} + \cdots + g_0, \qquad (1)$$

the underscore denotes the vector of coefficients, that is,

$$\underline{G} = \begin{bmatrix} g_n \\ g_{n-1} \\ \vdots \\ g_0 \end{bmatrix}. \qquad (2)$$

When a polynomial coefficient vector such as $\underline{G}$ is used in an equation requiring a vector of larger dimension, $\underline{G}$ should be understood to include additional zero coefficients corresponding to higher powers of s:

$$G(s) = 0s^{n+k} + \cdots + 0s^{n+1} + g_n s^n + + \cdots + g_0. \qquad (3)$$

Similarly, the symbol $\underline{0}$ denotes a zero vector of whatever dimension is appropriate.

An overbar denotes the Toeplitz matrix

$$\overline{G} = \begin{bmatrix} \underline{G} & 0 & & 0 \\ 0 & \underline{G} & & 0 \\ 0 & 0 & \dots & \vdots \\ \vdots & \vdots & & 0 \\ 0 & 0 & & \underline{G} \end{bmatrix}. \qquad (4)$$

The number of columns is determined by the context in which the matrix appears.

I denotes an identity matrix of whatever dimension is appropriate. Superscript T denotes transposition.

B. Regression Vector Construction

The regression vector construction is shown graphically in Figure 1. In the Figure, u_p and y_p denote, respectively, the input and output of an n^{th} order SISO plant. The signal r is the exogenous command input.

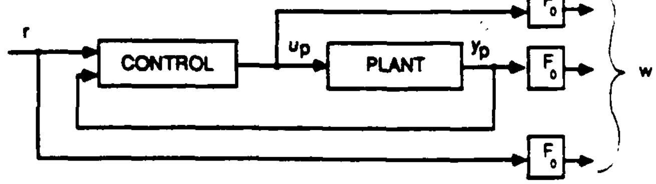

Figure 1

Regression Vector Construction

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Let $\Lambda(s)$ be any chosen Hurwitz polynomial of degree $N \geq n$, and define

$$F_0 := \frac{\begin{bmatrix} s^N \\ s^{N-1} \\ \vdots \\ 1 \end{bmatrix}}{\Lambda(s)}, \quad F := \begin{bmatrix} F_0 & 0 & 0 \\ 0 & F_0 & 0 \\ 0 & 0 & F_0 \end{bmatrix} \tag{5}$$

$$w_u := F_0 u_P, \quad w_y := F_0 y_P, \quad w_r := F_0 r \tag{6}$$

$$w := \begin{bmatrix} w_u \\ w_y \\ w_r \end{bmatrix} = F \begin{bmatrix} u_P \\ y_P \\ r \end{bmatrix}. \tag{7}$$

The signal $w(t)$ will be called the "regression vector."

III. GENERAL ADAPTIVE CONTROL STRUCTURE

A. Ideal Tuned System

In the analysis of the actual adaptive system, it is useful to compare its behavior with that of a hypothetical "ideal tuned system." By "ideal," we mean a system without nonparametric uncertainty. By "tuned," we mean a system with the controller gains initialized so as to achieve the specified goal for the particular plant.

In this subsection, we briefly state the general structure as it applies to the ideal tuned system.

1. The Three Structure Equations

The tuned ideal system is described by

$$\textit{Plant}: \quad \Theta_{P0}^T w^* = 0 \tag{8a}$$

$$\textit{Control}: \quad \Theta_{C0}^T w^* = 0 \tag{8b}$$

$$\textit{Goal}: \quad \Theta_{G0}^T w^* = 0 \tag{8c}$$

where

$$\Theta_{P0} = \Theta_{P1} + C_P \Theta_P \tag{9a}$$

$$\Theta_{C0} = \Theta_{C1} + C_C \Theta_P \tag{9b}$$

$$\Theta_{G0} = \Theta_{G1} + C_G \Theta_P \tag{9c}$$

Here w^* is the regression vector which would be produced by the ideal tuned system (given r), Θ_{P1}, Θ_{C1}, and Θ_{G1} are all known constant vectors, and C_P, C_C, and C_G are all known constant matrices. Θ_P is the vector of unknown plant parameters, which can have smaller dimension that the total plant parameter vector Θ_{P0} (that is, C_P can be nonsquare). We have chosen to allow the goal to be plant-dependent, as shown in equation (9c).

Remark: Equation (9a) is chosen to have the form above for flexibility in representing the location of uncertain parameters within the plant dynamics, and (9c) is chosen for flexibility in specifying the goal. Any value for Θ_{P1}, C_P, Θ_{G1}, and C_G is allowed. However, (9b) is *not* a matter of choice; the form of (9b) and the values of Θ_{C1} and C_C are determined by (9a) and (9c), as will be shown later (Theorem 1).

2. Interrelationship Between Plant, Control, and Goal

Underlying the notion of "tuned" system is the understanding that the plant equation and the control equation together imply achievement of the goal, without any other assumptions regarding the excitation. This may be stated more precisely. The Laplace transform domain is appropriate for this LTI interrelationship.

Consider the scalar field S = { scalar convolution kernels having a rational Laplace transfrom representation }, and let scalar multiplication be convolution in the time domain. Let X be the linear space over S of all triplets $\{u_P, y_P, r\}$ such that each is a Lebesgue-measurable function.

The plant constraint (8a) is

$$\Theta_{P0}^T F \begin{bmatrix} u_P \\ y_P \\ r \end{bmatrix} = 0. \tag{10a}$$

The control constraint (8b) is

$$\Theta_{C0}^T F \begin{bmatrix} u_P \\ y_P \\ r \end{bmatrix} = 0. \tag{10b}$$

It is necessary that, together, (10a) and (10b) imply the goal

$$\Theta_{G0}^T F \begin{bmatrix} u_P \\ y_P \\ r \end{bmatrix} = 0 \tag{10c}$$

without any further restrictions on $\{u_P, y_P, r\} \in X$.

Note that (10a) and (10b) are each a linear constraint on X over S, which together must imply the third linear constraint (10c). That is, the third constraint lies in the subspace spanned by the first two, hence there must exist scalars $\hat{a}, \hat{b} \in S$ such that

$$\hat{a}\Theta_{P0}^T F + \hat{b}\Theta_{C0}^T F = \Theta_{G0}^T F. \tag{11}$$

Viewed in the Laplace transform domain, (11) is an equality of rational transfer function vectors. The denominators of the transforms of $\hat{a}$ and $\hat{b}$ can be cleared to yield

$$a\Theta_{P0}^T F + b\Theta_{C0}^T F = c\Theta_{G0}^T F \tag{12}$$

for some polynomials a, b, and c.

An immediate consequence is the following theorem.

Theorem 1 : Given the plant and goal parameterizations (9a) and (9c), the interrelationship requirement (12) implies (9b) with

$$\Theta_{C1} = (\bar{b}^T\bar{b})^{-1}\bar{b}^T(-\bar{a}\Theta_{P1} + \bar{c}\Theta_{G1})$$
$$C_C = (\bar{b}^T\bar{b})^{-1}\bar{b}^T(-\bar{a}C_P + \bar{c}C_G)$$

where $\bar{a}, \bar{b}, \bar{c}$ are Toeplitx matrices corresponding to the polynomials a, b, c of (12).

The appendix contains a proof.

Remark: Theorem 1 shows precisely how the location of the plant parameter uncertainty within the plant dynamics (9a) and the structure of the dependence of the goal on the uncertain plant parameters (9c) determine the structure of the controller. The adjustable control coefficients must be located so as to be equivalent to (9b) with the known parts given by Theorem 1. Formally, the adjustable controller coefficients Θ_C must be such that there exists a mapping Q from the Θ_C space onto the Θ_P space ($\Theta_P = Q(\Theta_C)$, where all Θ_P are reachable by choice of Θ_C). (For uniqueness of the controller gains in the ideal case, the map Q must also be invertible.) Furthermore, the controller parameterization must be equivalent to

$$(\Theta_{C1}^T + Q(\Theta_C)^T C_C^T)w = 0 \tag{13}$$

where the values of Θ_{C1} and C_C are given by Theorem 1.

Another consequence of (12) is the following theorem.

Theorem 2 : The interrelationship equation (12) implies

$$aC_P^T F + bC_C^T F = cC_G^T F$$

The appendix contains a proof.

Theorem 2 provides another interrelationship expression, derived from the first, which will be useful later in the derivation of the stability analysis setting.

Returning now to (12), one can also deduce the following. Let p be any Hurwitz polynomial of sufficiently high degree that

$$a' = a/p \tag{14a}$$

$$b' = b/p \tag{14b}$$

$$c' = c/p \tag{14c}$$

are all proper. Then (12) implies that

$$a'\Theta_{P0}^T w + b'\Theta_{C0}^T w = c'\Theta_{G0}^T w \tag{15a}$$

for $w = F\begin{bmatrix} u \\ y \\ r \end{bmatrix}$ and *any* u, y, r (which need *not* satisfy (8a,b,c)).

B. Actual (nonideal) System

In the actual system, the controller gains are not necessarily tuned; incomplete prior knowledge of the plant parameters prevents initial perfect tuning. Furthermore, the plant may contain non-parametric uncertainty, which was neglected for the idealized system.

1. Three Structure Equations

The actual system is described by

$$\textit{Plant}: \quad \Theta_{P0}^T w = d = \Delta v \tag{16a}$$

$$\textit{Control}: \quad \theta_{C0}^T w = 0 \tag{16b}$$

$$\textit{Goal}: \quad \theta_{G0}^T w = e_0 \tag{16c}$$

where Θ_{P0} is a constant vector, θ_{C0} is the time-varying estimate of the desired parameter vector Θ_{C0}, θ_{G0} is a possibly-time-varying parameter describing the plant-dependent goal, and w is the regression vector produced by the actual system. It is assumed that v is known (constructable) given w, and that Δ is an unknown dynamical operator, constituting nonparametric plant uncertainty.

As in the case of the ideal system, the parameter vectors of equations (16) can be described in terms of their known and unknown parts:

$$\Theta_{P0} = \Theta_{P1} + C_P \Theta_P \tag{17a}$$

$$\theta_{C0}(t) = \Theta_{C1} + C_C \theta_P(t) \tag{17b}$$

$$\theta_{G0}(t) = \Theta_{G1} + C_G \theta_P(t) \tag{17c}$$

Here $\theta_P(t)$ is the plant parameter estimate corresponding to the adjustable controller parameters. When the controller parameters are identified directly (direct adaptive control), the plant parameter estimate $\theta_P(t)$ is implicit; the adaptive system might not solve for θ_P, but $\theta_{C0}(t)$ of equation (16b) must still satisfy (17b) for some $\theta_P(t)$, at each time t.

2. Construction of the Plant Input

The actual system is the system one implements, as opposed to the conceptual ideal system described earlier. Here we explicitly state the plant input definition which is implicit in (16b), so that the control law implementation is apparent. We also introduce the final constraint on the structures we address, namely that the plant input be properly defined by (16b). This constraint is nothing more than the always-present requirement that a controller implementation does not involve the construction of improper transfer functions or the closing of ill-posed loops.

First let us consider the special case in which the first element of the vector θ_{C0} is never zero. Later we will transform a more general case into the form of this special case.

Let the filter F of equation (5) be factored into a strictly proper part F_P and a real gain matrix D:

$$F = F_P + D \tag{18}$$

Then

$$w = \begin{bmatrix} F_P u_P \\ w_y \\ w_r \end{bmatrix} + \begin{bmatrix} Du_P \\ 0 \\ 0 \end{bmatrix} =: w_P + \begin{bmatrix} D \\ 0 \\ 0 \end{bmatrix} u_P \tag{19}$$

$$\theta_{C0}^T w = 0 \quad \text{is equivalent to} \quad \theta_{C0}^T w_P + \theta_{C0}^T \begin{bmatrix} D \\ 0 \\ 0 \end{bmatrix} u_P = 0. \tag{20}$$

For notational convenience, denote the time-varying real scalar c_u by

$$\theta_{C0}^T \begin{bmatrix} D \\ 0 \\ 0 \end{bmatrix} =: c_u. \tag{21}$$

The scalar c_u is nonzero by the assumption that the leading coefficient of θ_{C0} is nonzero. It follows from (20) that u_P is well-defined by

$$u_P = \frac{\theta_{C0}^T w_P}{c_u}. \tag{22}$$

Equations (21) and (22) should be used in *implementing* the adaptive controller, although we will continue to use the equivalent equation (16b) for analysis purposes.

The above derivation of the plant input involved the special case where the first element of θ_{C0} is known to be non-zero. We now show how other cases can be transformed into this special case.

Note that all control laws of the form

$$\theta_{C0}^T w = 0 \tag{23}$$

are representations of the equation

$$\frac{K_1}{\Lambda} u_P - \frac{K_2}{\Lambda} y_P - \frac{K_3}{\Lambda} r = 0 \tag{24}$$

where K_i are arbitrary polynomials in s, and the transfer functions are proper. That is, equation (24) is equivalent to

$$[\, \underline{K}_1^T \;\mid\; -\underline{K}_2^T \;\mid\; -\underline{K}_3^T \,] w = 0. \tag{25}$$

As explained in the section on Notation, a coeffiecient vector representing a polynomial can have leading zero coefficients for compatibility of dimensions. As a consequence, the $\underline{K}_i$ of (25) can have leading coefficients which are zero, in general.

Note that for constant parameters, (24) is equivalent to

$$u_P = \frac{K_2}{K_1} y_P + \frac{K_3}{K_1} r \tag{26}$$

In non-adaptive control, the control law (26) can be implemented only if the degree of K_1 is at least as great as the degree of K_2 and K_3 We require that for θ_{C0} frozen at any instant in time, the resulting LTI control law corresponds to (26) with the degree of K_1 always n_1, and the degree of K_2 and K_3 always $\leq n_1$, with n_1 arbitrary but independent of time.

Let Λ_1 be a polynomial such that the degree of $K_1\Lambda_1$ equals N, the degree of Λ (equation (5). Then the control law (26) is equivalent to

$$\frac{K_1\Lambda_1}{\Lambda} u_P - \frac{K_2\Lambda_1}{\Lambda} y_P - \frac{K_3\Lambda_1}{\Lambda} r = 0 \tag{27}$$

which in turn is equivalent to

$$[\,K_1^T \mid -K_2^T \mid -K_3^T\,]\begin{bmatrix}\bar{\Lambda}_1^T & 0 & 0\\ 0 & \bar{\Lambda}_1^T & 0\\ 0 & 0 & \bar{\Lambda}_1^T\end{bmatrix} w = 0. \tag{28}$$

Now, for compatibility of dimensions in the above matrix multiplications, no zeros are augmented to the K_1 vector. That is, the first coefficient of K_1 is nonzero by the assumption on the degree of the product $K_1\Lambda_1$.

Allowing the coefficients of the K_i of (28) to be adjustable, one obtains an adjustable controller parameterization. Defining

$$\theta_{new} := \begin{bmatrix}K_1\\ -K_2\\ -K_3\end{bmatrix}, \quad w_{new} := \begin{bmatrix}\bar{\Lambda}_1^T & 0 & 0\\ 0 & \bar{\Lambda}_1^T & 0\\ 0 & 0 & \bar{\Lambda}_1^T\end{bmatrix} w, \tag{29}$$

one finds that equation (28) is $\theta_{new}^T w_{new} = 0$, with the coefficient vector having a nonzero leading coefficient. As a consequence, the earlier technique for explicitly solving for u_P (culminating in equation (22)) applies.

3. Summary

The class of systems we are addressing are all those which, under idealizing approximations, can be represented by (8a,b,c) and (9a,c), and in actuality can be represented by (16a,b,c) and (17a,c). Two other minor (but important) requirements are added:

(1) For an adaptive system to make sense, the goal must be reachable by suitable choice of the controller parameters. This is the interrelationship requirement, which was shown to be equivalent to a linear constraint, which in turn leads to the requirement that the controller be equivalent to (9b) and (17b).

(2) For an adaptive system to be implementable, the implicit definition of the plant input in (17b) must be obtainable with an explicit construction involving only proper transfer functions. This is not a limitation of our representation; it is a basic requirement of all control systems.

IV. DERIVATION OF THE IDENTIFICATION SETTING

In this section, a certain error signal is defined in the same general terms as the general structure. This error signal is constructable in real time using known quantities, and captures the parameter error information in precisely the form required by the robust identification theory of [4], [2], and [3].

Let f_i, $i = 1, 2, \ldots, M$ be any M user-chosen stable LTI filters, and recall the definitions of a', b', c' from equation (14). For each filter f_i, one can construct the signal

$$e_i = \theta_{C0}^T(-f_i b' w) + \theta_{C0}^T(f_i c' w). \tag{30}$$

Let the parameter error be denoted by $\phi_P(t) = \theta_P(t) - \Theta_P$, where θ_P is the (possibly implicit) plant parameter estimate.

Theorem 3 : $e_i = \phi_P^T w_i' + d_i'$

where $w_i' = f_i(-b' C_C^T w + c' C_C^T w)$

and $d_i' = f_i a' d = \Delta f_i a' v = \Delta v_i'$.

The appendix contains a proof.

In the above, e_i', w_i', and v_i' are all constructable quantities. The theorem states that the error signal e_i is the product of the unknown parameter vector and a known "regression vector" w_i', plus a "noise" term. This is the setup of classical linear regression theory, dating back to Gauss [1] and treated in depth for the stochastic noise case in modern textbooks, such as [9]. In our formulation, d_i' is not characterized by a stochastic distribution. Instead, we represent d_i' as the output of an uncertain dynamical operator, intended to capture the nonparametric uncertainty of the plant dynamics. This class of uncertainty is addressed in the robust parameter adjustment theory of [4], [2], and [3], which applies to precisely the equations of Theorem 3.

While the identification problem has been derived for the general structure, and given by equation (30) and Theorem 3 above, the actual application and analysis of the robust identification theory in the context of the general structure has not yet been performed.

V. SPECIAL CASES

A. Plant Equation

The general structure equations include the plant equations (8a), (9a) for the ideal tuned case, and (16a) and (17a) for the nonideal case. These descriptions include, as a special case, the conventional polynomial coefficient uncertainty addressed by most adaptive control theory, as well as a broad class of other plants not previously addressed, as the following examples illustrate.

1. Common Example

Consider

$$y_P = \left[\frac{N_P}{D_P}\right] u_P, \tag{31}$$

where N_P and D_P are polynomials with unknown coefficients. One can always choose one coefficient at will. For example, let us choose the denominator polynomial to be monic (i.e., leading coefficient equals one). Then the polynomials are

$$N_P = n_m s^m + n_{m-1} s^{m-1} + \ldots + n_0 \tag{32}$$

$$D_P = s^n + d_{n-1} s^{n-1} + \ldots + d_0 \tag{33}$$

where the n_i and d_i are the unknown coefficients.

Choosing any Hurwitz polynomial Λ for the definition of F (equation (5)), one can rewrite (31) as

$$-\frac{N_P}{\Lambda}u_P + \frac{D_P}{\Lambda}y_P = 0 \tag{34}$$

which is equivalent to

$$-n_m\left[\frac{s^m}{\Lambda}u_P\right] - n_{m-1}\left[\frac{s^{m-1}}{\Lambda}u_P\right] - \ldots - n_0\left[\frac{1}{\Lambda}u_P\right]\ldots + \left[\frac{s^n}{\Lambda}y_P\right] + \ldots + d_0\left[\frac{1}{\Lambda}y_P\right] = 0. \tag{35}$$

With the definition of w from (7), (35) is

$$(\Theta_{P1}^T + \Theta_P^T C_P^T)w = 0 \tag{36a}$$

$$\Theta_{P1}^T = [\,0\ 0\ \ldots\ 0\ 1\ 0\ \ldots\ 0\,] \tag{36b}$$

$$\Theta_P^T := [\,-n_m\ \ -n_{m-1}\ \ldots\ -n_0\ \ d_{n-1}\ \ldots\ d_0\,] \tag{36c}$$

$$C_P^T := \begin{bmatrix}0 & I & 0 & 0 & 0\\ 0 & 0 & 0 & I & 0\end{bmatrix}. \tag{36d}$$

for 0 and I matrices of the appropriate dimension.

Thus the most commonly studied case of a transfer function with unknown polynomial coefficients falls within the general structure representation, using an appropriate choice of Θ_{P1}, Θ_P, and C_P.

2. Common Example Plus Stable Factor Perturbation

Consider the plant

$$y_p = \left[\frac{D_P}{\Lambda} + \Delta_2 W_2\right]^{-1}\left[\frac{N_p}{\Lambda} + \Delta_1 W_1\right] \tag{37}$$

Note that with $\Delta_1 = \Delta_2 = 0$, this plant is the same as that of equation (31). Alternatively, one can allow Δ_1 and Δ_2 to represent additional nonparametric uncertainty, in recognition of the fact that parametric descriptions never exactly capture the input/output behavior of physical systems.

Now, denoting

$$v = \begin{bmatrix} W_1 & 0 \\ 0 & -W_2 \end{bmatrix} \begin{bmatrix} u_p \\ y_p \end{bmatrix} \tag{38}$$

$$\Delta = [\ \Delta_1 \ \ \Delta_2\ ] \tag{39}$$

and repeating the algebraic derivation for the common example above, one obtains

$$(\Theta_{p1}^T + \Theta_p^T C_p^T) w = \Delta v \tag{40}$$

with the same definitions of Θ_{p1}, Θ_p, and C_p as in (36). Thus a plant with numerator and denominator polynomial coefficient uncertainty and a nonparametric stable factor perturbation can be represented within the general structure.

3. More General Example

The above parametric uncertainty corresponds to complete pole and zero uncertainty. This high degree of uncertainty is frequently conservative; plant uncertainty often admits a lower-degree parameterization. Fortunately, the general structure allows for reduced uncertainty order. For example, consider the parameterization illustrated in Figure 2. In the figure,

$$M_1(s) = \begin{bmatrix} M_{11}(s) \ M_{12}(s) \\ M_{21}(s) \ M_{22}(s) \end{bmatrix} \tag{35}$$

is a known, stable, proper, linear time-invariant transfer function matrix. The parameter vector Θ_p in this figure can have fewer elements than the corresponding vector in (36).

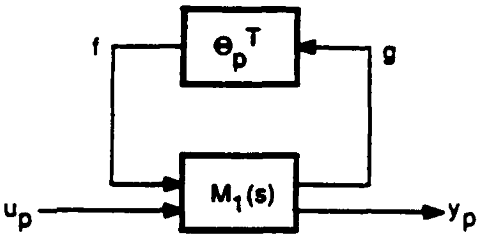

Figure 2

Plant Parameterization

Lemma 1 : Without loss of generality, we may assume that M_{21}^{-1} is stable and proper. (Proof in appendix.)

Now, through some algebraic manipulation, we get

$$\begin{bmatrix} f \\ g \end{bmatrix} = \begin{bmatrix} -M_{21}^{-1}M_{22} & M_{21}^{-1} \\ M_{12} - M_{11}M_{21}^{-1}M_{22} & M_{11}M_{21}^{-1} \end{bmatrix} \begin{bmatrix} u_p \\ y_p \end{bmatrix}. \tag{42}$$

Extracting a common denominator, Λ, yields

$$\begin{bmatrix} f \\ g \end{bmatrix} = \begin{bmatrix} \frac{N_{11}}{\Lambda} & \frac{N_{12}}{\Lambda} \\ \frac{N_{21}}{\Lambda} & \frac{N_{22}}{\Lambda} \end{bmatrix} \begin{bmatrix} u_p \\ y_p \end{bmatrix}. \tag{43}$$

Letting $\underline{N}_{ij}$ denote the suitably arranged matrix of coefficients of the numerator polynomials, $f = \Theta_p^T g$ (from Figure 2) yields

$$([\ \underline{N}_{11}^T \ \ \underline{N}_{12}^T \ \ 0\ ] + \Theta_p^T[\ \underline{N}_{21}^T \ \ \underline{N}_{22}^T \ \ 0\ ])w = 0. \tag{44}$$

This corresponds to the general structure equation (8a) and (9a). Here the choices of Θ_{p1} and C_p are shown explicitly. The dimension of the uncertain vector Θ_p is reduced by the presence of the known matrix C_p, which is calculated from the interconnection structure M.

The plant parameterization of Figure 2 includes, as a special case, the complete parameter uncertainty of (36), by choosing

$$M_{11} = \begin{bmatrix} \Omega \\ F_0 \end{bmatrix},\ M_{12} = \begin{bmatrix} F_0 \\ \Omega \end{bmatrix},\ M_{21} = 1,\ M_{22} = 0. \tag{45}$$

As the above example illustrates, the general structure includes a broad class of systems with parametric uncertainty embedded within the dynamics of the system in a more complicated fashion than the commonly studied polynomial-coefficient-uncertainty case.

4. More General Example Plus Nonparametric Perturbation

Consider the system of Figure 3 , with

$$M_2 = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \\ M_{31} & M_{32} \end{bmatrix} \tag{46}$$

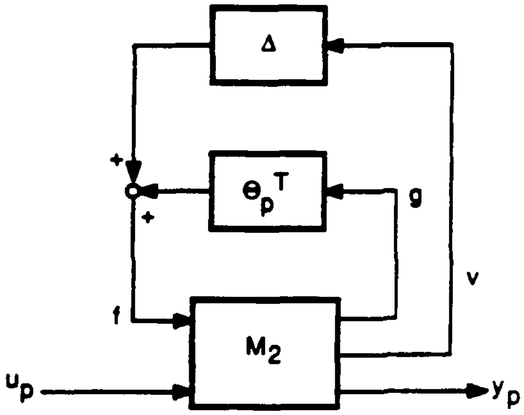

Figure 3

Plant Parameterization with Perturbation

Through some algebraic manipulation, one obtains

$$\begin{bmatrix} f \\ g \\ v \end{bmatrix} = \begin{bmatrix} -M_{31}^{-1}M_{32} & M_{31}^{-1} \\ M_{12} - M_{11}M_{31}^{-1}M_{32} & M_{11}M_{31}^{-1} \\ M_{22} - M_{21}M_{31}^{-1}M_{32} & M_{21}M_{31}^{-1} \end{bmatrix} \begin{bmatrix} u_p \\ y_p \end{bmatrix} \tag{47}$$

$$= \begin{bmatrix} \frac{N_{11}^T}{\Lambda} & \frac{N_{12}^T}{\Lambda} \\ \frac{N_{21}^T}{\Lambda} & \frac{N_{22}^T}{\Lambda} \\ \frac{N_{31}^T}{\Lambda} & \frac{N_{33}^T}{\Lambda} \end{bmatrix} \begin{bmatrix} u_p \\ y_p \end{bmatrix}. \tag{48}$$

Applying this to $f = \Theta_p^T g + \Delta v$ (from Figure 3) yields

$$([\ \underline{N}_{11}^T \ \ \underline{N}_{12}^T \ \ 0\ ] + \Theta_p^T[\ \underline{N}_{21}^T \ \ \underline{N}_{22}^T \ \ 0\ ])w = \Delta v \tag{49}$$

where v is a constructable quantity. Equation (49) is equivalent to the general structure equations (16a) and (9a). Thus any system containing uncertain parameters and nonparametric uncertainty interconnected in the fairly general manner shown in Figure 3 can be represented by the general structure equations.

B. Controller Equation

This section contains a discussion of the flexibility afforded by the controller equation (8b) and (16b).

The flexibility in defining the plant input by (16b) can be seen by noting that

$$u_p = \frac{\Lambda}{\Lambda} u_p = [\, \Delta^T \mid \underline{0}^T \mid \underline{0}^T \,] w, \tag{50}$$

(the last equality is due to equations (5) through (7)), so that equation (16b) defines u_p to be

$$u_p = \left[\theta_{C0}^T + [\, \Delta^T \mid \underline{0}^T \mid \underline{0}^T \,]\right] w. \tag{51}$$

Thus by selecting the value of θ_{C0}, one is designing the feedback gains (modulo a simple known translation of coordinates) from the signal vector w to the plant input.

The signal w contains N smoothed derivatives of the plant input and output, and the command input. This allows the controller considerable freedom. In standard model reference control structures (for example, [8]), the controller feeds back smoothed derivatives of the plant input and output, and, in our notation, a constrained combination of elements of w_r, namely

$$k \Delta^T w_r \tag{52}$$

where k is the adjustable gain. Thus the controller gain placement of [8] is a special case of (16b) of the general structure.

Another special case is the adaptive state feedback controller of [6]. There, in the implementation of the adjustable feedback controller, the plant input is defined to be an adjustable parameter vector times smoothed derivatives of the plant input and output plus, again, a constrained combination of the elements of w_r. Thus state feedback control structures are a special case of the general structure control equation.

The general structure equation also offers additional flexibility not afforded by the special cases mentioned above. In the general structure, the gains associated with the elements of w_r need not be constrained to allow only a single parametric degree of freedom, as in (52). Instead, a command-shaping filter may be adaptively designed.

Concrete examples of controller equations will also appear in the following section, in conjunction with examples of goal equations.

C. Goal Equation

The general structure equations include the goal equations (8c), (9c) for the ideal tuned case, and (16c) and (17c) for the nonideal case. These descriptions include, as a special case, the conventional model following goal, as well as unconventional goals in adaptive control, such as loop shaping. This is illustrated by the following examples.

1. Model Following

The goal of model reference adaptive control is to have the plant output y_p respond to commands r as a chosen reference model would respond, that is, to achieve

$$y_p = \frac{N_M}{D_M} r \tag{53}$$

where N_M and D_M are chosen model polynomials of degree m and n, respectively, where m and n are the degrees of the plant numerator and denominator polynomials. The above is equivalent to

$$\frac{D_M}{\Lambda} y_p - \frac{N_M}{\Lambda} r = 0 \tag{54}$$

which in turn is equivalent to the goal equation

$$[\, \underline{0}^T \mid \underline{D}_M^T \mid -\underline{N}_M^T \,] w = 0 \tag{55}$$

An interesting alternative (for reasons to be seen later) is the following. Let N_X be a chosen Hurwitz polynomial of the form

$$N_X(s) = n_{x,n-m-1} s^{n-m-1} + n_{x,n-m-2} s^{n-m-2} + \ldots + 1 \tag{56}$$

with all of the coefficients $n_{x,i}$ allowed to be arbitrarily small. Then N_X is approximately equal to 1 except at high frequencies, and in fact for the case of relative degree one plants ($n-m = 1$), N_X is exactly equal to 1 at all frequencies. Consider now the *approximate* model matching goal

$$y_p = \frac{N_M}{D_M + (N_X - 1) D_P} r \tag{57}$$

where the idealized plant is assumed to be given by equation (31). With some prior knowledge of the maximum magnitude of $D_P(s)$, N_X can be chosen sufficiently close to 1 so that the response of y_P to r is essentially equal to the model response over all frequencies of interest.

It can be verified that the closed loop response of equation (57) is achieved if and only if

$$u_p = \frac{N_M}{N_P N_X} r + \frac{D_P - D_M}{N_P N_X} y_P \tag{58}$$

which in turn is equivalent to

$$\frac{N_P N_X \Lambda_1}{\Lambda} u_P - \frac{N_M \Lambda_1}{\Lambda} r + \frac{(-D_P + D_M)\Lambda_1}{\Lambda} y_p = 0 \tag{59}$$

where Λ_1 as a degree-one factor of Λ, Λ is chosen to have degree $N = n$, and (without loss of generality) D_P and D_M are assumed to be monic. Recalling equation (34) for the idealized plant, (59) is equivalent to

$$\frac{N_P (N_X - 1)}{\Lambda} u_P - \frac{N_M}{\Lambda} r + \frac{-D_P + D_M}{\Lambda} y_p = 0 \tag{60}$$

which in turn is equivalent to

$$[\, \underline{N}_P^T \overline{(N_X - 1)}^T \mid -\underline{D}_P^T + \underline{D}_M^T \mid -\underline{N}_M^T \,] w = 0. \tag{61}$$

with overbar denoting a Toeplitz matrix as described in the section on notation. This, then, may be taken as the goal equation $\Theta_{G0}^T w = 0$.

The reason that (61) is an interesting alternative to (55) is that the scalars a, b, c relating the plant, control, and goal equations in equation (12) are $a = c = \Lambda_1$, $b = 1$. This can be seen by noting that the ideal plant corresponds to the one-by-three transfer function matrix

$$\Theta_{P0}^T F = \begin{bmatrix} \frac{-N_P}{\Lambda} & \frac{D_P}{\Lambda} & 0 \end{bmatrix} \tag{62}$$

and the tuned controller equation (59) corresponds to the transfer function matrix

$$\Theta_{C0}^T F = \begin{bmatrix} \frac{N_P N_X \Lambda_1}{\Lambda} & \frac{(-D_P + D_M)\Lambda_1}{\Lambda} & \frac{-N_M \Lambda_1}{\Lambda} \end{bmatrix} \tag{63}$$

and the goal equation (61) corresponds to the transfer function matrix

$$\Theta_{G0}^T F = \begin{bmatrix} \frac{N_P (N_X - 1)}{\Lambda} & \frac{D_M}{\Lambda} & \frac{-N_M}{\Lambda} \end{bmatrix}. \tag{64}$$

Equation (12) follows, with $a = c = \Lambda_1$, $b = 1$.

The utility of $a = c = \Lambda_1$, $b = 1$ is twofold. First, these scalars are independent of the unknown plant parameters and are thus known *a priori*. This is important in the set-up of the identification problem with nonparametric dynamics present, as can be seen from the construction of e_i (equation (30)) and w_i' (Theorem 3). Second, the simple values of a, b, c lead to a simplification of the expressions in Theorem 1 and Theorem 3.

Engineering remarks: For all real systems, unmodeled dynamics make the control of high frequ behavior impossible. Thus the above low frequency approximation to model following does not actually represent a degradation in model following performance, in a practical sense. Moreover, at those frequencies where N_X is not equal to 1, the presence of N_X gives the feedback loop added rolloff (compared to $N_X = 1$), which improves the high-frequency robustness margins.

Theoretical remarks: The theoretical purest may achieve N_X equal to 1, and simultaneously achieve $a = c = \Lambda_1$ and $b = 1$ by simply always assuming the plant to be relative degree 1. The weighting function on the nonparametric dynamics (W_1 of equation (37)) can be chosen such that the assumption that N_P has degree n-1 is always valid to within the tolerances allowed by the nonparametric dynamics.

2. Loop Shaping

Consider again the ideal plant of equation (31). Suppose that the denominator polynomial can be factored into

$$D_P = D_{P1} D_{P2} \tag{65}$$

where D_{P1} is Hurwitz. Assume also that the numerator N_P is Hurwitz (right-half-plane zeros of the actual (nonideal) plant must be absorbed into the nonparametric uncertainty). Let Λ of equation (5) be represented in factored form as

$$\Lambda = \Lambda_1 \Lambda_2 \tag{66}$$

where Λ_2 has degree $N-m$ and Λ_1 has degree m (recall $N \geq n$).

Consider the following loop shape:

$$L = \frac{1}{D_{P2}\Lambda_2}. \tag{66}$$

In the above, the classical convention of negative feedback has been used, that is, a negative sign is omitted from the loop gain in representing the loop shape. The above loop shape can be obtained by the control law

$$u_P = \frac{D_{P1}}{N_P \Lambda_2}(r - y_p) \tag{67}$$

which is equivalent to

$$\frac{N_P \Lambda_2}{\Lambda} u_P + \frac{D_{P1}}{\Lambda} y_P - \frac{D_{P1}}{\Lambda} r = 0 \tag{68}$$

which is equivalent to

$$[\ \underline{N}_{P1}^T \overline{\Lambda}_2^T \ \mid \ \underline{D}_{P1}^T \ \mid \ -\underline{D}_{P1}^T\ ] w = 0. \tag{69}$$

This, then, may be taken as the tuned controller equation.

Application of the control law (67) produces a response

$$y_P = \frac{1}{D_{P2}\Lambda_2 + 1} r = \frac{D_{P1}}{D_P \Lambda_2 + D_{P1}} r \tag{70}$$

which is equivalent to

$$[\ \underline{0}^T \ \mid \ \underline{D}^P \overline{\Lambda}_2^T + \underline{D}_{P1} \ \mid \ -\underline{D}_{P1}^T\ ] w = 0. \tag{71}$$

This, then, may be taken as the goal equation.

Following steps analogous to those of equations (62) through (64), one finds that the interrelationship equation (12) is satisfied using known scalar polynomials:

$$\begin{aligned} a &= \Lambda_2 \\ b &= 1 \\ c &= 1. \end{aligned} \tag{72}$$

Remark 1: If the plant is known to be open loop stable, the plant denominator can be partitioned into $D_{P1} = D_P$, $D_{P2} = 1$, in which case the tuned closed loop response is

$$\frac{y_P}{r} = \frac{1}{\Lambda_2 + 1} \tag{73}$$

which is known *a priori*. Alternatively, if the plant is not stable but the poles of D_{P2} known (to within the tolerance allowed by nonparametric uncertainty), then the ideal tuned command response is again known *a priori* (equation (70)). Finally, if the poles which are not to be cancelled are *not* known well *a priori*, then the tuned system command response is also not known well *a priori*.

Remark 2: The loop shape reveals several important properties of a control system. A large loop gain (compared to 1) at low frequencies results in insensitivity to disturbances and nonparametric plant dynamics. A small loop gain (compared to 1) at high frequencies results in good robustness margins, allowing large nonparametric plant dynamics at these frequencies. Where the loop gain is *near* 1, a loop phase lag *much less than* 180 degrees results in good damping, proper transient response, and robustness to small nonparametric plant dynamics. Consequently, an on-line adaptive design of a preselected loop shape may prove practically desirable in some cases.

VI. CONCLUSIONS

This paper has presented a general adaptive control structure which provides flexibility. For example, the plant parameter uncertainty can be imbedded in the plant dynamics to reflect the physical source of the uncertainty as understood by the control system engineer. Likewise, there is flexibility in defining the goal of the adaptation, with loop-shaping and model-reference goals being among the alternatives.

As theoretical progress is made in robust stability and performance analysis of adaptive control systems, those results posed in the framework of the general structure of this paper will inherit its degree of validity and applicability.

The convenience of a single simple representation for a broad class of adaptive systems also provides us with a new opportunity: we can now address "best structure" issues. Adaptive control theoreticians can proceed to formulate various performance objectives, and seek the particular adaptive system structure within the general structure representation which best accomplishes the objective. In effect, we have taken the first step in optimizing over the set of alternative structures; we have defined the set of alternatives in a simple and explicit fashion.

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APPENDIX

Proof of Lemma 1: One can factor $M_{21}(s)$ into $(H(s)/G(s))\hat{M}_{21}(s)$ where $H(s)$ contains the right-half-plane zeros of $M_{21}(s)$, $G(s)$ is Hurwitz, G/H is proper, and $\hat{M}_{21}^{-1}$ is stable and proper. Then defining $f_{new} = (H/G)f$, $g_{new} = (H/G)g$, one obtains

$$\begin{bmatrix} g_{new} \\ y_P \end{bmatrix} = \begin{bmatrix} M_{11} & (H/G)M_{12} \\ \hat{M}_{21} & M_{22} \end{bmatrix}, \quad f_{new} = \Theta_P^T g_{new}.$$

□

Proof of Theorem 1:

Equating the numerator coefficients of (12), one obtains the following real-coefficient equaltiy:

$$\overline{a}\Theta_{P0} + \overline{b}\Theta_{C0} = \overline{c}\Theta_{G0}. \tag{74}$$

Expanding this in terms of the tuned plant and goal parameterizations of equation (9) yields

$$\overline{a}(\Theta_{P1} + C_P\Theta_P) + \overline{b}\Theta_{C0} - \overline{c}(\Theta_{G1} + C_G\Theta_P) = 0. \tag{75}$$

The least-squares solution for Θ_{C0} is

$$\Theta_{C0} = (\bar{b}^T \bar{b})^{-1} \bar{b}^T \left[(-\bar{a}\Theta_{P1} + \bar{c}\Theta_{G1}) + (-\bar{a}C_P + \bar{c}C_G)\Theta_P \right] \tag{76}$$

For $\Theta_{C0} = \Theta_{C1} + C_C \Theta_P$ (equation (9)) to be consistent with (76) for fixed Θ_{C1} and C_C, irrespective of the value of Θ_P, it is necessary that the least squares solution be an exact solution, and that (equating (9) to (76))

$$\Theta_{C1} = (\bar{b}^T \bar{b})^{-1} \bar{b}^T (-\bar{a}\Theta_{P1} + \bar{c}\Theta_{G1}) \tag{77}$$

$$C_C = (\bar{b}^T \bar{b})^{-1} \bar{b}^T (-\bar{a}C_P + \bar{c}C_G) \tag{78}$$

which is the statement of the theorem. Added remark: the least squares solution for the ideal tuned controller must *exactly* solve (75). Thus, substituting the solution for Θ_{C0} into (75), and requiring the equation to be valid for all Θ_P, one obtains

$$\bar{b}(\bar{b}^T \bar{b})^{-1} \bar{b}^T (\bar{a}\Theta_{P1} + \bar{c}\Theta_{G1}) = (\bar{a}\Theta_{P1} + \bar{c}\Theta_{G1}) \tag{79}$$

$$\bar{b}(\bar{b}^T \bar{b})^{-1} \bar{b}^T (\bar{a}C_P + \bar{c}C_G) = (\bar{a}C_P + \bar{c}C_G) \tag{80}$$

□

Proof of Theorem 2: the equality (12) is valid as Θ_P varies. Therefore, equating partial derivatives (with respect to each element of Θ_P) of each side yields the theorem. □

Proof of Theorem 3: Let

$$\phi_{C0} := \theta_{C0} - \Theta_{C0} = C_C \phi_P \tag{81}$$

$$\phi_{G0} := \theta_{G0} - \Theta_{G0} = C_G \phi_P \tag{82}$$

$$\tilde{w} := w - w^*. \tag{83}$$

Equations (8a) and (16a) imply $\Theta_{P0}^T \tilde{w} = d$. (84)

Equations (81) and (16b) imply $\Theta_{C0}^T w + \phi_{C0}^T w = 0$. (85)

Equations (8b) and (85) imply $\Theta_{C0}^T \tilde{w} + \phi_{C0}^T w = 0$. (86)

Equation (16c) implies $\Theta_{G0}^T w + \phi_{G0}^T w = e_0$. (87)

Equations (8c) and (87) imply $\Theta_{G0}^T \tilde{w} + \phi_{G0}^T w = e_0$ (88)

Multiplying (88) by c' from (14c) yields

$$c'\Theta_{G0}^T \tilde{w} + c'\phi_{G0}^T w = c'e_0. \tag{89}$$

Note that

$$\tilde{w} = F \begin{bmatrix} \tilde{u}_P \\ \tilde{y}_P \\ \tilde{r} \end{bmatrix}$$

where $\tilde{u}_P = u_P - u^*_P$, $\tilde{y}_P = y_P - y^*_P$, $\tilde{r} = r - r^* (= 0)$, so equation (15) applies to $\tilde{w}$. Therefore, applying (15) to the first term of (89) yields

$$a'\Theta_{P0}^T \tilde{w} + b'\Theta_{C0}^T \tilde{w} + c'\phi_{G0}^T w = c'e_0. \tag{90}$$

Applying (84) and (86) to (90) yields

$$a'd - b'\phi_{C0}^T w + c'\phi_{G0}^T w = c'e_0. \tag{91}$$

The usual error augmentation ([7], [5]) is based on the manipulation: $f\phi w = f\theta w - f\Theta w = f\theta w - \Theta f w$ (since constants commute with LTI filters), hence $f\phi w + \theta f w - f\theta w = \phi f \theta$ where the left hand side of this last expression is the "augmented" version of $f\phi w$. Now, incorporating the filter f_i and applying the usual error augmentation procedure above to move the time varying quantities ϕ_{C0} and ϕ_{G0} to the left of the scalar transfer functions, (91) becomes

$$f_i c' e_0 + \theta_{C0}^T(-f_i b' w) + f b'(\theta_{C0}^T w) + \theta_{G0}^T(f c' w) - f c'(\theta_{G0}^T w)$$
$$= \phi_P^T(-f_i b' C_C^T w + f_i c' C_G^T w). \tag{92}$$

Now $\theta_{G0}^T w = e_0$ from (16a) leads to a cancellation of two terms on the left hand side of (92). $\theta_{C0}^T w = 0$ from equation (16b) causes another term to drop out. The result is the statement of the Theorem, which was to be proved. □

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